

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

- 1 | 1. (Currently amended) A method for improving computation efficiency in  
2 | solving using a computer system to solve a global optimization problem specified  
3 | by a function  $f$  and a set of equality constraints, the method comprising:  
4 |       receiving a representation of the function  $f$  and the set of equality  
5 | constraints  $q_i(\mathbf{x}) = 0$  ( $i=1, \dots, r$ ) at the computer system, wherein  $f$  is a scalar  
6 | function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ ;  
7 |       storing the representation in a memory within the computer system;  
8 |       performing an interval global optimization process to compute guaranteed  
9 | bounds on a globally minimum value of the function  $f(\mathbf{x})$  subject to the set of  
10 | equality constraints with improved computation efficiency;  
11 |       wherein performing computations during interval global optimization  
12 | process involves using a special-purpose interval arithmetic unit for interval  
13 | computations;  
14 |       wherein performing the interval global optimization process involves,  
15 |               applying term consistency to the set of equality constraints  
16 |               over a subbox  $\mathbf{X}$ , and  
17 |               excluding portions of the subbox  $\mathbf{X}$  that can be shown to  
18 |               violate any of the equality constraints;  
19 |       wherein applying term consistency involves:  
20 |               symbolically manipulating an equation within the computer  
21 | system to solve for a term,  $g(x_i)$ , thereby producing a modified

22 equation  $g(x_j) = h(\mathbf{x})$ , wherein the term  $g(x_j)$  can be analytically  
 23 inverted to produce an inverse function  $g^{-1}(y)$ ;  
 24 substituting the subbox  $\mathbf{X}$  into the modified equation to  
 25 produce the equation  $g(X'_j) = h(\mathbf{X})$ ;  
 26 solving for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and  
 27 intersecting  $X'_j$  with the interval  $X_j$  to produce a new  
 28 subbox  $\mathbf{X}^+$ ;  
 29 wherein the new subbox  $\mathbf{X}^+$  contains all solutions of the  
 30 equation within the subbox  $\mathbf{X}$ , and wherein the size of the new  
 31 subbox  $\mathbf{X}^+$  is less than or equal to the size of the subbox  $\mathbf{X}$ .

1       2. (Original) The method of claim 1, wherein performing the interval  
 2 global optimization process involves:  
 3       preconditioning the set of equality constraints through multiplication by an  
 4 approximate inverse matrix  $\mathbf{B}$  to produce a set of preconditioned equality  
 5 constraints;  
 6       applying term consistency to the set of preconditioned equality constraints  
 7 over the subbox  $\mathbf{X}$ ; and  
 8       excluding portions of the subbox  $\mathbf{X}$  that can be shown to violate any of the  
 9 preconditioned equality constraints.

1       3. (Original) The method of claim 1, wherein performing the interval  
 2 global optimization process involves:  
 3       keeping track of a least upper bound  $f\_bar$  of the function  $f(\mathbf{x})$ ;  
 4       unconditionally removing from consideration any subbox for which  
 5  $\inf(f(\mathbf{x})) > f\_bar$ ;  
 6       applying term consistency to the inequality  $f(\mathbf{x}) \# f\_bar$  over the subbox  $\mathbf{X}$ ;  
 7 and

8           excluding portions of the subbox **X** that violate the inequality.

1           4. (Canceled)

1           5. (Original) The method of claim 1, wherein performing the interval  
2 global optimization process involves:  
3           applying box consistency to the set of equality constraints  $q_i(\mathbf{x}) = 0$   
4 ( $i=1, \dots, r$ ) over the subbox **X**; and  
5           excluding portions of the subbox **X** that violate the set of equality  
6 constraints.

1           6. (Original) The method of claim 1, wherein performing the interval  
2 global optimization process involves:  
3           evaluating a first termination condition;  
4           wherein the first termination condition is TRUE if a function of the width  
5 of the subbox **X** is less than a pre-specified value,  $\epsilon_X$ , and the absolute value of the  
6 function,  $f$ , over the subbox **X** is less than a pre-specified value,  $\epsilon_F$ ; and  
7           if the first termination condition is TRUE, terminating further splitting of  
8 the subbox **X**.

1           7. (Original) The method of claim 1, wherein performing the interval  
2 global optimization process involves performing an interval Newton step on the  
3 John conditions.

1           8. (Currently amended) A computer-readable storage medium storing  
2 instructions that when executed by a computer system cause the computer system  
3 | to perform a method for improving computation efficiency in solving using a

4 | ~~computer system to solve~~ a global optimization problem specified by a function  $f$   
 5 | and a set of equality constraints, the method comprising:  
 6 |       receiving a representation of the function  $f$  and the set of equality  
 7 | constraints  $q_i(\mathbf{x}) = 0$  ( $i=1, \dots, r$ ) at the computer system, wherein  $f$  is a scalar  
 8 | function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ ;  
 9 |       storing the representation in a memory within the computer system;  
 10 |       performing an interval global optimization process to compute guaranteed  
 11 | bounds on a globally minimum value of the function  $f(\mathbf{x})$  subject to the set of  
 12 | equality constraints with improved computation efficiency;  
 13 |       wherein performing computations during interval global optimization  
 14 | process involves using a special-purpose interval arithmetic unit for interval  
 15 | computations;  
 16 |       wherein performing the interval global optimization process involves,  
 17 |               applying term consistency to the set of equality constraints  
 18 |               over a subbox  $\mathbf{X}$ , and  
 19 |               excluding portions of the subbox  $\mathbf{X}$  that can be shown to  
 20 |               violate any of the equality constraints;  
 21 |       wherein applying term consistency involves:  
 22 |               symbolically manipulating an equation within the computer  
 23 |               system to solve for a term,  $g(x_j)$ , thereby producing a modified  
 24 |               equation  $g(x_j) = h(\mathbf{x})$ , wherein the term  $g(x_j)$  can be analytically  
 25 |               inverted to produce an inverse function  $g^{-1}(y)$ ;  
 26 |               substituting the subbox  $\mathbf{X}$  into the modified equation to  
 27 |               produce the equation  $g(X'_j) = h(\mathbf{X})$ ;  
 28 |               solving for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and  
 29 |               intersecting  $X'_j$  with the interval  $X_j$  to produce a new  
 30 |               subbox  $\mathbf{X}^+$ ;

31                   wherein the new subbox  $X^+$  contains all solutions of the  
32                   equation within the subbox  $X$ , and wherein the size of the new  
33                   subbox  $X^+$  is less than or equal to the size of the subbox  $X$ .

1           9. (Original) The computer-readable storage medium of claim 8, wherein  
2 performing the interval global optimization process involves:  
3           preconditioning the set of equality constraints through multiplication by an  
4 approximate inverse matrix  $B$  to produce a set of preconditioned equality  
5 constraints;  
6           applying term consistency to the set of preconditioned equality constraints  
7 over the subbox  $X$ ; and  
8           excluding portions of the subbox  $X$  that can be shown to violate any of the  
9 preconditioned equality constraints.

1           10. (Original) The computer-readable storage medium of claim 8, wherein  
2 performing the interval global optimization process involves:  
3           keeping track of a least upper bound  $f\_bar$  of the function  $f(x)$ ;  
4           unconditionally removing from consideration any subbox for which  
5  $\inf(f(x)) > f\_bar$ ;  
6           applying term consistency to the inequality  $f(x) \# f\_bar$  over the subbox  $X$ ;  
7 and  
8           excluding portions of the subbox  $X$  that violate the inequality.

1           11. (Canceled)

1           12. (Original) The computer-readable storage medium of claim 8, wherein  
2 performing the interval global optimization process involves:

3           applying box consistency to the set of equality constraints  $q_i(\mathbf{x}) = 0$   
4   ( $i=1, \dots, r$ ) over the subbox  $\mathbf{X}$ ; and  
5           excluding portions of the subbox  $\mathbf{X}$  that violate the set of equality  
6   constraints.

1           13. (Original) The computer-readable storage medium of claim 8, wherein  
2   performing the interval global optimization process involves:  
3           evaluating a first termination condition;  
4           wherein the first termination condition is TRUE if a function of the width  
5   of the subbox  $\mathbf{X}$  is less than a pre-specified value,  $\varepsilon_X$ , and the absolute value of the  
6   function,  $f$ , over the subbox  $\mathbf{X}$  is less than a pre-specified value,  $\varepsilon_F$ ; and  
7           if the first termination condition is TRUE, terminating further splitting of  
8   the subbox  $\mathbf{X}$ .

1           14. (Original) The computer-readable storage medium of claim 8, wherein  
2   performing the interval global optimization process involves performing an  
3   interval Newton step on the John conditions.

1           15. (Currently amended) An apparatus that improves computation  
2   efficiency in solving solves a global optimization problem specified by a function  
3    $f$  and a set of equality constraints, the apparatus comprising:  
4           a receiving mechanism that is configured to receive a representation of the  
5   function  $f$  and the set of equality constraints  $q_i(\mathbf{x}) = 0$  ( $i=1, \dots, r$ ), wherein  $f$  is a  
6   scalar function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ ;  
7           a memory for storing the representation;  
8           an optimizer that is configured to perform an interval global optimization  
9   process to compute guaranteed bounds on a globally minimum value of the

10 function  $f(\mathbf{x})$  subject to the set of equality constraints with improved computation  
 11 efficiency;  
 12 wherein the optimizer configured to perform computations during interval  
 13 global optimization process involves using a special-purpose interval arithmetic  
 14 unit for interval computations;  
 15 wherein the optimizer is configured to,  
 16 apply term consistency to the set of equality constraints  
 17 over a subbox  $\mathbf{X}$ , and to  
 18 exclude portions of the subbox  $\mathbf{X}$  that can be shown to  
 19 violate any of the equality constraints;  
 20 wherein while applying term consistency, the optimizer is configured to:  
 21 symbolically manipulate an equation to solve for a term,  
 22  $g(x_j)$ , thereby producing a modified equation  $g(x_j) = h(\mathbf{x})$ , wherein  
 23 the term  $g(x_j)$  can be analytically inverted to produce an inverse  
 24 function  $g^{-1}(y)$ ;  
 25 substitute the subbox  $\mathbf{X}$  into the modified equation to  
 26 produce the equation  $g(X'_j) = h(\mathbf{X})$ ;  
 27 solve for  $X'_j = g^{-1}(h(\mathbf{X}))$ ; and to  
 28 intersect  $X'_j$  with the interval  $X_j$  to produce a new  
 29 subbox  $\mathbf{X}^+$ ;  
 30 wherein the new subbox  $\mathbf{X}^+$  contains all solutions of the  
 31 equation within the subbox  $\mathbf{X}$ , and wherein the size of the new  
 32 subbox  $\mathbf{X}^+$  is less than or equal to the size of the subbox  $\mathbf{X}$ .

1 16. (Original) The apparatus of claim 15, wherein the optimizer is  
 2 configured to:

3 precondition the set of equality constraints through multiplication by an  
4 approximate inverse matrix **B** to produce a set of preconditioned equality  
5 constraints;  
6 apply term consistency to the set of preconditioned equality constraints  
7 over the subbox **X**; and to  
8 exclude portions of the subbox **X** that can be shown to violate any of the  
9 preconditioned equality constraints.

1 17. (Original) The apparatus of claim 15, wherein the optimizer is  
2 configured to:  
3 keep track of a least upper bound  $f\_bar$  of the function  $f(\mathbf{x})$ ;  
4 unconditionally remove from consideration any subbox for which  
5  $\inf(f(\mathbf{x})) > f\_bar$ ;  
6 apply term consistency to the inequality  $f(\mathbf{x}) \# f\_bar$  over the subbox **X**;  
7 and to  
8 exclude portions of the subbox **X** that violate the inequality.

1 18. (Canceled)

1 19. (Original) The apparatus of claim 15, wherein the optimizer is  
2 configured to:  
3 apply box consistency to the set of equality constraints  $q_i(\mathbf{x}) = 0$  ( $i=1, \dots, r$ )  
4 over the subbox **X**; and to  
5 exclude portions of the subbox **X** that violate the set of equality  
6 constraints.

1 20. (Original) The apparatus of claim 15, wherein the optimizer is  
2 configured to:



3           evaluate a first termination condition;  
4           wherein the first termination condition is TRUE if a function of the width  
5 of the subbox **X** is less than a pre-specified value,  $\varepsilon_X$ , and the absolute value of the  
6 function,  $f$ , over the subbox **X** is less than a pre-specified value,  $\varepsilon_F$ ; and to  
7           terminate further splitting of the subbox **X** if the first termination  
8 condition is TRUE

1           21. (Original) The apparatus of claim 15, wherein the optimizer is  
2 configured to perform an interval Newton step on the John conditions.